CHAPTER 9

TAKEOFF AND LANDING PERFORMANCE

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EQUATIONS

$$D = \left(C_{D_p} + \frac{C_L^2}{\pi\, e\, AR} \right) q\, S$$ (Eq 9.13) 9.10

$$T_{ex} = T - \left(C_{D_p} + \frac{C_L^2}{\pi\, e\, AR} \right) q\, S - \mu\left(W - C_L\, q\, S \right)$$ (Eq 9.14) 9.10

$$\frac{dT_{ex}}{dC_L} = \left(\frac{2\, C_L}{\pi\, e\, AR} \right) q\, S + \mu\left(q\, S \right)$$ (Eq 9.15) 9.10

$$C_{L_{Opt}} = \frac{\mu\, \pi\, e\, AR}{2}$$ (Eq 9.16) 9.10

$$S_2 = \int_{\text{Lift off}}^{50\text{ ft}} (T - D)\, dS = \frac{W}{2g} \left(V_{50}^2 - V_{TO}^2 \right) + 50\, W$$ (Eq 9.17) 9.12

$$S_2 = \frac{W \left(\frac{V_{50}^2 - V_{TO}^2}{2g} + 50 \right)}{(T - D)_{Avg}}$$ (Eq 9.18) 9.12

$$V_{TO_w} = V_{TO} - V_w$$ (Eq 9.19) 9.13

$$S_{1_w} = \frac{W\, V_{TO_w}^2}{2\, g\, T_{ex_{Avg_w}}}$$ (Eq 9.20) 9.13

$$S_{1_{Std}} = \frac{W \left(V_{TO_w} + V_w \right)^2}{2\, g\, T_{ex_{Avg}}}$$ (Eq 9.21) 9.13

$$S_{1_{Std}} = S_{1_w} \frac{T_{ex_{Avg_w}}}{T_{ex_{Avg}}} \left(1 + \frac{V_w}{V_{TO_w}} \right)^2$$ (Eq 9.22) 9.13

$$S_3 = \frac{W\left(\frac{V_{TD}^2 - V_{50}^2}{2g} - 50\right)}{(T - D)_{Avg}}$$ (Eq 9.32) 9.19

$$S_4 = \int_{Touchdown}^{Stop} \left[T\text{-}D\text{-}\mu(W - L) \right] dS = \frac{1}{2}\ \frac{W}{g}\left(0 - V_{TD}^2\right)$$ (Eq 9.33) 9.19

$$S_4 = \frac{-W\ V_{TD}^2}{2g\left[\ T - D - \mu(W - L)\right]_{Avg}}$$ (Eq 9.34) 9.20

$$S_{3_{Std}} = S_{3_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{\left(2 + \frac{E_h}{E_h + 50}\right)} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)^{\left(\frac{E_h}{E_h + 50}\right)}$$ (Eq 9.35) 9.23

$$E_h = \frac{V_{50}^2 - V_{TD}^2}{2g}$$ (Eq 9.36) 9.23

$$S_{4_{Std}} = S_{4_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^2 \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)$$ (Eq 9.37) 9.23

V_w = Wind Velocity cos (Wind Direction Relative To Runway) (Eq 9.38) 9.28

$$\sigma = 9.625\ \frac{P_a}{T_a}$$ (Eq 9.39) 9.29

$$V_{TD_w} = V_{TD} - V_w$$ (Eq 9.40) 9.30

$$S_{4_{Std}} = S_{4_w} \left(1 + \frac{V_w}{V_{TD}}\right)^{1.85}$$ (Eq 9.41) 9.30

$$S_{4_{Std}} = \frac{S_{4_{SL}}}{\left(1 - \frac{2\, g\, S_{4_{SL}}}{V_{TD}^2}\sin\theta\right)}$$

(Eq 9.42) 9.31

CHAPTER 9

TAKEOFF AND LANDING PERFORMANCE

9.1 INTRODUCTION

Field takeoff and landing tests are important portions of the flight test program for any aircraft. Generally, during the course of a flight test program, all takeoffs and landings are recorded for data purposes. Also, a number of test flights may be devoted entirely to takeoff and landing tests in various configurations including, aborted takeoffs, crosswind operations, wet/icy runway operations, landings in various configurations, and field arrested landings. All are accomplished at various gross weights.

The primary emphasis of this chapter is to discuss the conventional takeoff and landing (CTOL) performance of fixed wing aircraft supported primarily by aerodynamic forces rather than engine thrust. Discussion of short takeoff and landing (STOL) performance is limited to two sections of the chapter which discuss methods of shortening the takeoff and landing distance.

More than most other tests, takeoffs and landings are affected by factors which cannot be accurately measured nor properly compensated for. Only estimates of the capabilities of the aircraft are possible within rather broad limits, relying on a statistical average of numerous takeoffs and landings to minimize residual errors.

For purposes of this chapter, Naval Air Systems Command Specification, AS-5263, “Guidelines For Preparation Of Standard Aircraft Characteristics Charts And Performance Data Piloted Aircraft (Fixed Wing)”, is used to establish the criteria for takeoff and landing performance tests (Table 9.1).

Table 9.1

AS-5263 REQUIREMENTS

	Takeoff	Landing
Speeds (1)	V_{TO} at 1.1 times speed represented by 90% $C_{L_{max_{TO}}}$ $V_{CL_{50}}$ at ≥ 1.2 V_{s_T}	V_L at ≥ 1.1 V_{s_L} $V_{L_{50}}$ at ≥ 1.2 V_{s_L}
Distance	Takeoff ground roll plus distance to climb to 50 ft	Distance from 50 ft to touchdown plus landing roll
Rolling Coefficient	0.025	
Braking Coefficient		0.30

Note:

[1] Other criteria may apply also

Where:

$C_{L_{max_{TO}}}$	Maximum lift coefficient, takeoff configuration	
$V_{CL_{50}}$	Climb speed at 50 ft	kn
V_L	Landing airspeed	ft/s, kn
$V_{L_{50}}$	Landing speed at 50 ft	kn
V_{s_L}	Stall speed, landing configuration, power off	kn
V_{s_T}	Stall speed, transition configuration, power off, flaps down, gear up	kn
V_{TO}	Takeoff ground speed	ft/s

The Federal Aviation Regulations (FAR) Part 23 and Part 25 establish different takeoff and landing criteria than AS-5263. With Department of the Navy acquiring off-the-shelf FAA certified aircraft, a review and understanding of the FAR is required before evaluating these aircraft for military missions.

9.2 PURPOSE OF TEST

The purpose of these tests include:

1. Development/verification of pilot takeoff and landing techniques appropriate for the test aircraft.

2. Develop flight manual data including:
    a. Normal ground roll takeoff distance (time/fuel).
    b. Distance, time, and fuel from liftoff to climb intercept.
    c. Minimum (short field) ground roll takeoff distance (time/fuel).
    d. Obstacle clearance takeoff distance (time/fuel).
    e. Takeoff speed.
    f. Speed/distances for checking takeoff acceleration.
    g. Maximum refusal speed.
    h. Emergency braking velocity.
    i. Effects of runway condition.
    j. Landing speed.
    k. Landing ground roll distance.
    l. Limit braking velocity for landing.

9.3 THEORY

9.3.1 TAKEOFF

The evaluation of takeoff performance can be examined in two phases, the ground and air phase. The ground phase begins at brake release, includes rotation, and terminates when the aircraft becomes airborne. The air phase is the portion of flight from leaving the ground until reaching an altitude of 50 ft. In the case where stabilizing at a constant climb speed before reaching 50 ft is possible, the air phase is divided into a transition phase and a steady state climb phase (Figure 9.1).

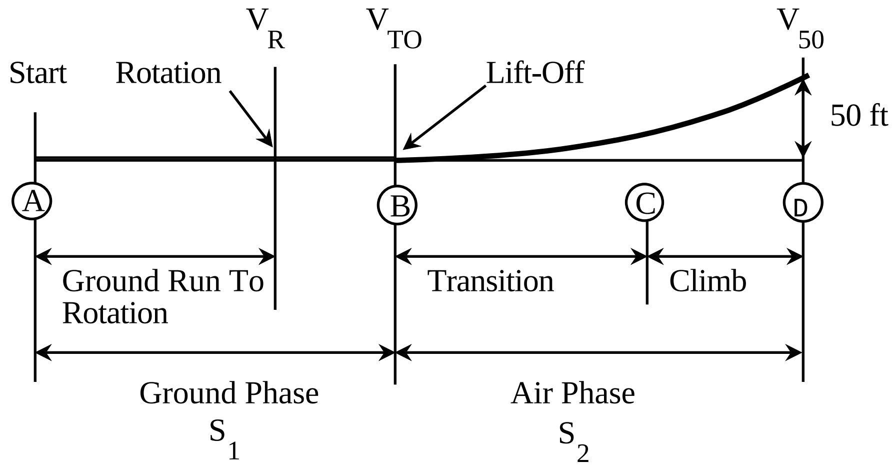


Figure 9.1
TAKEOFF PATH

Where:

S_1	Takeoff distance, brake release to lift off	ft
S_2	Takeoff distance, lift off to 50 ft	ft
V_{50}	Ground speed at 50 ft reference point	ft/s
V_R	Rotation airspeed	kn
V_{TO}	Takeoff ground speed	ft/s

Since lift off occurs almost immediately after or during rotation for most high performance aircraft, the ground phase is considered one distance (S_1) (Figure 9.1, A to B). Also, for most high performance aircraft, the transition to a steady climb speed is not completed before reaching 50 ft, even for a maximum climb angle takeoff. Therefore, the air phase is considered as one distance (S_2) (Figure 9.1, B to D).

9.3.1.1 FORCES ACTING DURING THE GROUND PHASE

The forces acting on the aircraft during the takeoff ground roll are shown in figure 9.2.

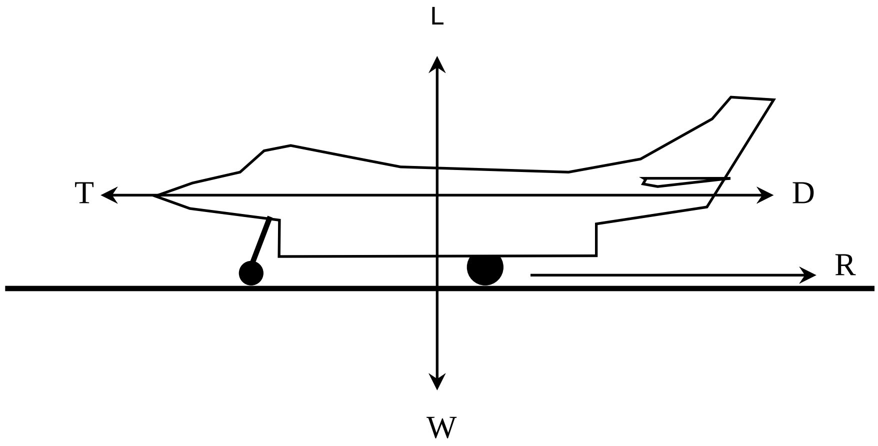


Figure 9.2
FORCES ACTING ON AN AIRCRAFT DURING TAKEOFF

In addition to the usual forces of lift, weight, thrust, and drag, an aircraft on takeoff roll is affected by an additional resistance force (R) which includes wheel bearing friction, brake drag, tire deformation, and energy absorbed by the wheels as they increase rotational speed. This force becomes smaller as lift increases and the weight-on-wheels is reduced. This resistance force can be expressed as:

$$R = \mu\ (W - L) \tag{Eq 9.1}$$

Typical values for the coefficient of friction (μ) are shown in Table 9.2.

Table 9.2

COEFFICIENT OF FRICTION VALUES

Surface	μ – Typical Values	
	Rolling, Brakes Off Ground Resistance Coefficient	Brakes On Wheel Braking Coefficient
Dry Concrete/Asphalt	0.02 – 0.05	0.3 – 0.5
Wet Concrete/Asphalt	0.05	0.15 – 0.3
Icy Concrete/Asphalt	0.02	0.06 – 0.1
Hard Turf	0.05	0.4
Firm Dirt	0.04	0.3
Soft Turf	0.07	0.2
Wet Grass	0.08	0.2

The arrangement of forces in figure 9.2 assumes engine thrust is parallel to the runway. For aircraft with engines mounted at an angle, the horizontal component of thrust is not reduced significantly until the angle becomes quite large. The vertical component of thrust from inclined engines reduces the effective weight of the aircraft. The mass of the aircraft, however, must be computed using the actual aircraft weight.

Setting the work done equal to the change in energy produces:

$$\int_0^{S_1} \left[T - D - \mu(W - L) \right] dS = \frac{1}{2} \frac{W}{g} \left(V_{TO}^2 \right) \qquad \text{(Eq 9.2)}$$

Since none of the terms under the integral are constant during the takeoff roll, an exact evaluation is virtually impossible. The expression can be evaluated assuming the entire quantity remains constant at some average value. The integration is simplified and the expression becomes:

$$\left[T - D - \mu(W - L) \right]_{Avg} S_1 = \frac{1}{2} \frac{W}{g} \left(V_{TO}^2 \right) \qquad \text{(Eq 9.3)}$$

Solving for S_1:

$$S_1 = \frac{W V_{TO}^2}{2g \left[T - D - \mu (W - L) \right]_{Avg}} \qquad \text{(Eq 9.4)}$$

Where:

D	Drag	lb
g	Gravitational acceleration	ft/s^2
L	Lift	lb
μ	Coefficient of friction	
R	Resistance force	lb
S_1	Takeoff distance, brake release to lift off	ft
T	Thrust	lb
V_{TO}	Takeoff ground speed	ft/s
W	Weight	lb.

Examination of the individual forces shows the assumption to be reasonable:

1. The engine thrust can be expected to decrease slightly as speed increases. A jet engine may enter ram recovery prior to lift off and realize an increase in thrust over that at lower speed. Propeller thrust will decrease throughout the takeoff roll.

2. Aerodynamic lift and drag increase during the roll in direct proportion to the square of the airspeed. If the aircraft attitude is changed considerably at rotation, both lift and drag increase sharply.

3. The coefficient of friction and the aircraft gross weight remain nearly constant.

The variations in these forces during the takeoff roll are shown graphically in figure 9.3.

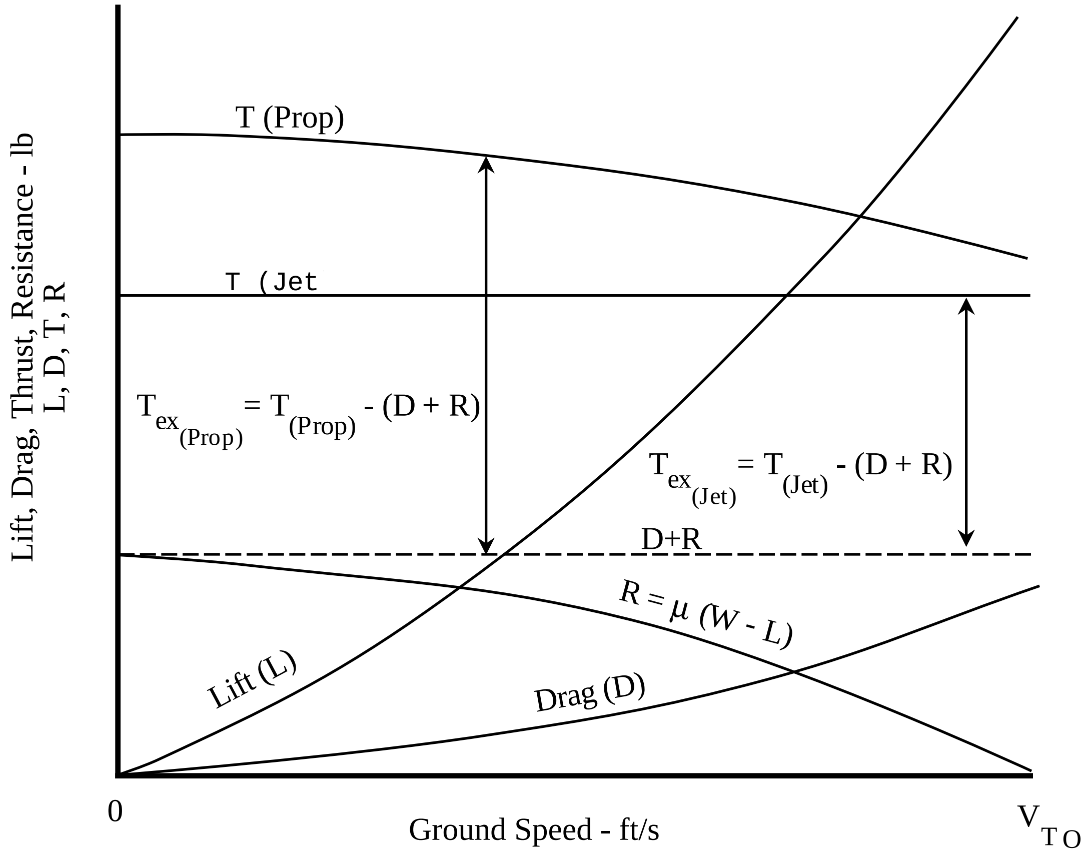


Figure 9.3
FORCE VERSUS VELOCITY

In general, the excess thrust (vector sum of T, D and R) at lift off is 80% of its initial value for a jet aircraft and 40% for a propeller aircraft. For both jets and props, test data has shown the use of the actual excess thrust at 0.75 V_{TO} as an average value for Eq 9.4 gives reasonable results.

9.3.1.2 SHORTENING THE TAKEOFF ROLL

Eq 9.4 shows ground roll can be shortened by lifting-off at a lower speed, since the distance increases with the square of the takeoff speed. Defining the takeoff test objectives as minimizing the ground roll, the aircraft should be lifted-off at $C_{L_{max}}$. However, the aerodynamic drag created by this technique may reduce excess thrust to an unacceptable level. In extreme cases, rotation to $C_{L_{max}}$ may reduce excess thrust with the result the aircraft will not accelerate or may even decelerate. If sufficient thrust is available to overcome the drag penalty, high lift slat and flap devices can provide a higher available lift coefficient.

A second approach to decreasing the takeoff distance (S_1) is increasing the thrust available either by operating the engine above its maximum rated power, such as by water injection or by use of an auxiliary engine such as JATO (Jet Assisted Takeoff). Thrust augmentation is of maximum value if it can be used throughout the takeoff roll. If augmentation is limited to a time shorter than required for takeoff, should the augmentation be used early or late in the ground roll? Since the energy gained equals the work done, limited augmentation is most efficient if used to maximize the work done. If the augmentation provides an increase in thrust (ΔT), for a fixed period of time (Δt), during which distance (ΔS) is traveled, then $\Delta S = V\Delta t$ and the work is:

$$\text{Work} = \Delta T \; V \; \Delta t \qquad \text{(Eq 9.5)}$$

Both ΔT and Δt are fixed by the limitations of the augmenting engine. The work done can be maximized if V is as large as possible. Therefore, for minimum ground roll, limited thrust augmentation should be used late, so it will burn out or reach its time limit just as the aircraft lifts-off.

Excess thrust during the takeoff roll is also dependent on aircraft angle of attack through both the drag term itself and the inclusion of lift in the wheel force term. If the optimum value of C_L is found, the best angle of attack to maximize excess thrust can be determined:

$$T_{ex} = T - D - \mu(W - L) \qquad \text{(Eq 9.6)}$$

$$q = \frac{1}{2}\rho V^2 \qquad \text{(Eq 9.7)}$$

$$D = C_D \; q \; S \qquad \text{(Eq 9.8)}$$

$$L = C_L \; q \; S \qquad \text{(Eq 9.9)}$$

$$C_D = C_{D_p} + C_{D_i} \qquad \text{(Eq 9.10)}$$

$$C_{D_i} = \frac{C_L^2}{\pi e AR} \qquad \text{(Eq 9.11)}$$

Substituting Eq 9.11 into Eq 9.10:

$$C_D = C_{D_p} + \frac{C_L^2}{\pi e AR} \qquad \text{(Eq 9.12)}$$

Substituting Eq 9.12 into Eq 9.8:

$$D = \left(C_{D_p} + \frac{C_L^2}{\pi e AR} \right) q S \qquad \text{(Eq 9.13)}$$

Substituting Eq 9.13 and Eq 9.9 into Eq 9.6:

$$T_{ex} = T - \left(C_{D_p} + \frac{C_L^2}{\pi e AR} \right) q S - \mu \left(W - C_L q S \right) \qquad \text{(Eq 9.14)}$$

Differentiating with respect to C_L:

$$\frac{dT_{ex}}{dC_L} = \left(\frac{2 C_L}{\pi e AR} \right) q S + \mu \left(q S \right) \qquad \text{(Eq 9.15)}$$

Setting the right side of Eq 9.15 equal to zero, the velocity term (q) drops out and the value of C_L for maximum excess thrust is constant and given by:

$$C_{L_{Opt}} = \frac{\mu \pi e AR}{2} \qquad \text{(Eq 9.16)}$$

Where:

AR	Aspect ratio	
C_D	Drag coefficient	
C_{D_i}	Induced drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
$C_{L_{Opt}}$	Optimum lift coefficient	
D	Drag	lb
e	Oswald's efficiency factor	
L	Lift	lb
μ	Coefficient of friction	
π	Constant	
q	Dynamic pressure	psf
ρ	Air density	slugs/ft^3
S	Wing area	ft^2
T	Thrust	lb
t	Time	s
T_{ex}	Excess thrust	lb
V	Velocity	ft/s
W	Weight	lb.

To achieve the shortest takeoff roll, a pilot establishes an angle of attack which corresponds to $C_{L_{Opt}}$ in Eq 9.16 and maintains $C_{L_{Opt}}$ until the speed permits rotation and lift off at $C_{L_{max}}$. In practice, however, this technique is seldom used because the dangers of over-rotating, lack of elevator or horizontal tail power, cross wind effects, or possible aircraft stability problems usually override any gain achieved. At the other extreme, since $C_{L_{Opt}}$ is quite small for most aircraft, an extremely long takeoff distance results if $C_{L_{Opt}}$ is held throughout the takeoff roll. As a practical matter, most aircraft are designed so that in the taxi attitude the wing is near the optimum angle of attack for minimizing the total resistance throughout takeoff. Therefore, most recommended takeoff techniques involve accelerating without changing attitude until the speed permits rotation and lift off at the maximum practical C_L available.

9.3.1.3 AIR PHASE

The equation for ground distance covered in climbing from lift off to 50 ft altitude is obtained in a manner similar to the ground roll equation except the resistance force no longer exists and a potential energy term must be included:

$$S_2 = \int_{\text{Lift off}}^{50\text{ ft}} (T - D)\, dS = \frac{W}{2g}\left(V_{50}^2 - V_{TO}^2\right) + 50\,W \qquad \text{(Eq 9.17)}$$

Assuming this quantity remains constant at some average value, the integration of Eq 9.17 becomes:

$$S_2 = \frac{W\left(\dfrac{V_{50}^2 - V_{TO}^2}{2g} + 50\right)}{(T - D)_{Avg}} \qquad \text{(Eq 9.18)}$$

Where:

D	Drag	lb
g	Gravitational acceleration	ft/s^2
S	Distance	ft
S_2	Takeoff distance, lift off to 50 ft	ft
T	Thrust	lb
V_{50}	Ground speed at 50 ft reference point	ft/s
V_{TO}	Takeoff ground speed	ft/s
W	Weight	lb.

To minimize the value of S_2 for a given weight, a constant speed climb is conducted at maximum excess thrust, while maximum excess thrust occurs at the speed for minimum drag, $\left.\frac{L}{D}\right|_{max}$, most aircraft lift off at an airspeed much slower than for $\left.\frac{L}{D}\right|_{max}$. As a practical matter, most high performance aircraft reach 50 ft within seconds while accelerating from lift off airspeed to climb airspeed.

9.3.1.4 TAKEOFF CORRECTIONS

9.3.1.4.1 WIND CORRECTION

The velocity in Eq 9.4 is ground speed at lift off, since this defines the energy level required. The aircraft flies according to the airspeed, which can be considerably different from ground speed in high winds. Since ground speed and true airspeed are equal in zero wind conditions, the ground speed required with wind, V_{TO_w} is:

$$V_{TO_w} = V_{TO} - V_w \tag{Eq 9.19}$$

V_w is positive for a head wind and includes only the component of wind velocity parallel to the takeoff direction. From Eq 9.4 and 9.6:

$$S_{1_w} = \frac{W\, V_{TO_w}^2}{2\, g\, T_{ex_{Avg_w}}} \tag{Eq 9.20}$$

The subscript, w, indicates a parameter in the wind environment. Substituting Eq 9.19 into Eq 9.20:

$$S_{1_{Std}} = \frac{W \left(V_{TO_w} + V_w \right)^2}{2\, g\, T_{ex_{Avg}}} \tag{Eq 9.21}$$

Dividing Eq 9.21 by Eq 9.20 and rearranging gives:

$$S_{1_{Std}} = S_{1_w} \frac{T_{ex_{Avg_w}}}{T_{ex_{Avg}}} \left(1 + \frac{V_w}{V_{TO_w}} \right)^2 \tag{Eq 9.22}$$

The difference in excess thrust due to wind is difficult to determine but it does have a significant effect on takeoff roll. For steady state winds of less than 10 kn, an empirical relationship has been developed that provides the following equation for the correction for head wind/tail wind components:

$$S_{1_{Std}} = S_{1_w}\left(1 + \frac{V_w}{V_{TO_w}}\right)^{1.85} \qquad \text{(Eq 9.23)}$$

Eq 9.23 does not account for gusts, which may have considerable effect if they occur near lift off speed. This is one of the reasons wind speed below 5 kn is required before takeoff data is accepted.

For the air phase, an exact determination of wind velocity is more difficult. The correction is simple, however, based on the fact that change in distance by wind is:

$$S_{2_{Std}} = S_{2_w} + \Delta S_2 \qquad \text{(Eq 9.24)}$$

Where:

D	Drag	lb
ΔS_2	Change in S_2, equal to t V_w	ft
g	Gravitational acceleration	ft/s^2
L	Lift	lb
S_1	Takeoff distance, brake release to lift off	ft
$S_{1_{Std}}$	Standard takeoff distance, brake release to lift off	ft
S_{1_w}	Takeoff distance, brake release to lift off, with respect to wind	ft
S_2	Takeoff distance, lift off to 50 ft	ft
$S_{2_{Std}}$	Standard takeoff distance, lift off to 50 ft	ft
S_{2_w}	Takeoff distance, lift off to 50 ft, with respect to wind	ft
T	Thrust	lb
t	Time	s
T_{ex}	Excess thrust	lb
$T_{ex_{Avg}}$	Average excess thrust	lb
$T_{ex_{Avg\,w}}$	Average excess thrust, with respect to wind	lb
V_{TO}	Takeoff ground speed	ft/s

V_{TO_W}	Takeoff ground speed with respect to wind	ft/s
V_W	Wind velocity	ft/s
W	Weight	lb.

9.3.1.4.2 RUNWAY SLOPE

The runway slope adds a potential energy term to Eq 9.3:

$$T_{ex_{Avg}} S_{1_{SL}} = \frac{1}{2} \frac{W}{g} V_{TO}^2 - W S_{1_{SL}} \sin\theta \tag{Eq 9.25}$$

The subscript, $_{SL}$, indicates a sloping runway parameter.

Solving for $S_{1_{SL}}$:

$$S_{1_{SL}} = \frac{W V_{TO}^2}{2 g \left(T_{ex_{Avg}} + W \sin\theta \right)} \tag{Eq 9.26}$$

Solving Eq 9.4 and 9.26 for average excess thrust, equating the results, and solving for S_1 produces an expression for a standard S_1:

$$S_{1_{Std}} = \frac{S_{1_{SL}}}{\left(1 - \frac{2g\, S_{1_{SL}}}{V_{TO}^2} \sin\theta \right)} \tag{Eq 9.27}$$

Where:

g	Gravitational acceleration	ft/s^2
θ	Runway slope angle	deg
S_1	Takeoff distance, brake release to lift off	ft
$S_{1_{SL}}$	Takeoff distance, brake release to lift off, sloping runway	ft
$S_{1_{Std}}$	Standard takeoff distance, brake release to lift off	ft
$T_{ex_{Avg}}$	Average excess thrust	lb

V_{TO}	Takeoff ground speed	ft/s
W	Weight	lb.

A fairly large slope is required before data is affected significantly. Low thrust-to-weight aircraft are affected more than high thrust-to-weight ratio aircraft.

9.3.1.4.3 THRUST, WEIGHT, AND DENSITY

Atmospheric conditions will affect the thrust available from the engine and will change the true airspeed required to fly a standard weight at a standard lift coefficient. As the weight changes, the airspeed required to fly at that C_L also changes. While an accurate analysis of these effects results in complex expressions, empirical relationships have been developed which provide reasonably accurate results.

For jet aircraft:

Ground phase:

$$S_{1_{Std}} = S_{1_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{2.3} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right) \left(\frac{T_{N_{Test}}}{T_{N_{Std}}}\right)^{1.3} \tag{Eq 9.28}$$

Air phase:

$$S_{2_{Std}} = S_{2_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{2.3} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)^{0.7} \left(\frac{T_{N_{Test}}}{T_{N_{Std}}}\right)^{1.6} \tag{Eq 9.29}$$

The accuracy of Eq 9.28 and Eq 9.29 depends on the determination of net thrust, T_N. Normally values developed from thrust stand data are used.

For turboprop aircraft with constant speed propellers the correction equations are:

Ground phase:

$$S_{1_{Std}} = S_{1_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{2.6} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)^{1.9} \left(\frac{N_{Test}}{N_{Std}}\right)^{0.7} \left(\frac{P_{a_{Test}}}{P_{a_{Std}}}\right)^{0.5} \qquad \text{(Eq 9.30)}$$

Air phase:

$$S_{2_{Std}} = S_{2_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{2.6} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)^{1.9} \left(\frac{N_{Test}}{N_{Std}}\right)^{0.8} \left(\frac{P_{a_{Test}}}{P_{a_{Std}}}\right)^{0.6} \qquad \text{(Eq 9.31)}$$

Where:

N_{Std}	Standard propeller speed	rpm
N_{Test}	Test propeller speed	rpm
$P_{a_{Std}}$	Standard ambient pressure	psf
$P_{a_{Test}}$	Test ambient pressure	psf
$S_{1_{Std}}$	Standard takeoff distance, brake release to lift off	ft
$S_{1_{Test}}$	Test takeoff distance, brake release to lift off	ft
$S_{2_{Std}}$	Standard takeoff distance, lift off to 50 ft	ft
$S_{2_{Test}}$	Test takeoff distance, lift off to 50 ft	ft
σ_{Std}	Standard density ratio	
σ_{Test}	Test density ratio	
$T_{N_{Std}}$	Standard net thrust	lb
$T_{N_{Test}}$	Test net thrust	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

9.3.1.5 PILOT TAKEOFF TECHNIQUE

Individual pilot technique can cause a greater variation in takeoff data than all other parameters combined. Some of the factors which significantly affect takeoff performance include:

1. Speed and sequence of brake release and power application.
2. The use of differential braking, nose wheel steering, or rudder deflection for directional control.
3. The number and amplitude of directional control inputs used.
4. Aileron/spoiler and elevator/horizontal tail position during acceleration.
5. Airspeed at rotation.
6. Pitch rate during rotation.
7. Angle of attack at lift off.

9.3.2 LANDING

The evaluation of landing performance can be examined in two phases, the air phase and the ground phase. The air phase starts at 50 ft above ground level and ends on touchdown. The ground phase begins at touchdown and terminates when the aircraft is stopped (Figure 9.4).

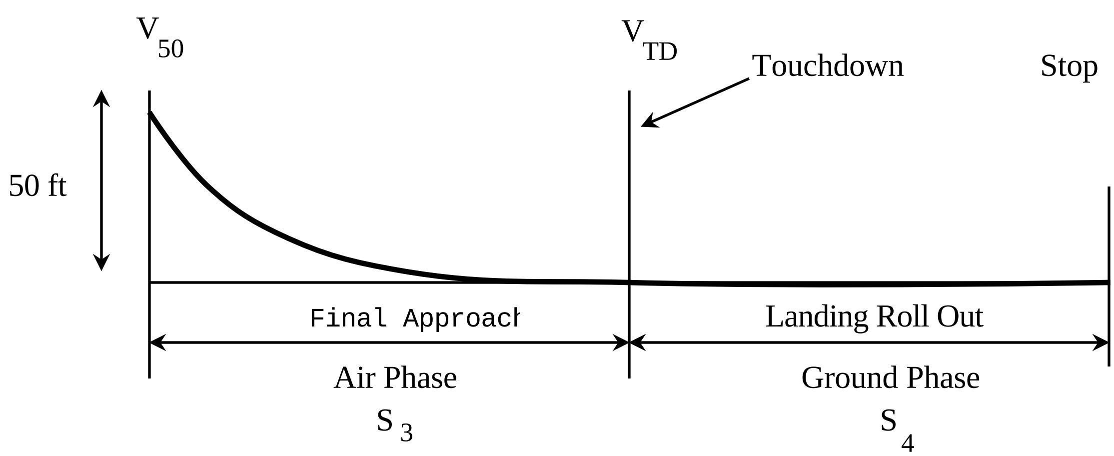


Figure 9.4
LANDING FLIGHT PHASES

Where:

S_3	Landing distance, 50 ft to touchdown	ft
S_4	Landing distance, touchdown to stop	ft
V_{50}	Ground speed at 50 ft reference point	ft/s
V_{TD}	Touchdown ground speed	ft/s

9.3.2.1 AIR PHASE

The equation governing the air distance on landing (S_3) is developed similarly to the takeoff equation:

$$S_3 = \frac{W\left(\frac{V_{TD}^2 - V_{50}^2}{2g} - 50\right)}{(T - D)_{Avg}} \qquad \text{(Eq 9.32)}$$

Where:

D	Drag	lb
g	Gravitational acceleration	ft/s^2
S_3	Landing distance, 50 ft to touchdown	ft
T	Thrust	lb
V_{50}	Ground speed at 50 ft reference point	ft/s
V_{TD}	Touchdown ground speed	ft/s
W	Weight	lb.

Examination of Eq 9.32 shows air distance is minimized if touchdown speed is maintained throughout the final descent (no flare) where $V_{TD} = V_{50}$ and a high drag/low thrust configuration (steep glide path) is used. The structural integrity of the aircraft becomes the limiting factor in this case.

9.3.2.2 FORCES ACTING DURING THE GROUND PHASE

The forces acting on an aircraft during the landing roll can be depicted similarly to those shown in figure 9.2 for takeoff. Low power settings and the increase in the coefficient of resistance due to brake application result in the excess thrust equation:

$$S_4 = \int_{\text{Touchdown}}^{\text{Stop}} \left[T\text{-}D\text{-}\mu(W - L) \right] dS = \frac{1}{2} \frac{W}{g} \left(0 - V_{TD}^2\right) \qquad \text{(Eq 9.33)}$$

When using an average value of the parameters the integration of Eq 9.33 becomes:

$$S_4 = \frac{-W\,V_{TD}^2}{2g\left[\,T - D - \mu(W - L)\,\right]_{Avg}} \qquad \text{(Eq 9.34)}$$

Where:

D	Drag	lb
g	Gravitational acceleration	ft/s^2
L	Lift	lb
μ	Coefficient of friction	
S	Distance	ft
S_4	Landing distance, touchdown to stop	ft
T	Thrust	lb
V_{TD}	Touchdown ground speed	ft/s
W	Weight	lb.

9.3.2.3 SHORTENING THE LANDING ROLL

Touchdown speed is one of the most important factors in the calculation of the distance required to stop. In addition to weight and speed at touchdown, landing roll can be influenced by all the factors in the excess thrust term. Thrust should be reduced to the minimum practical and, if available, reverse thrust should be employed as soon as possible after touchdown. The logic for early application of reverse thrust is the same as that for late use of time limited thrust augmentation on takeoff. Additional drag, whether from increased angle of attack (aerodynamic braking) or deployment of a drag chute, is most effective in the initial part of the landing roll for two reasons. Not only is a given force most effective at high speeds, but also the force itself is greater due to its dependence on V^2. Runway surface condition, as well as the mechanical design of the brakes themselves can cause the value of μ to vary over a considerable range. The assumption of an average excess thrust is reasonable as long as the attitude of the aircraft remains almost constant, but not if nose high aerodynamic braking is used after touchdown. Because aerodynamic braking is recommended to minimize landing roll for some aircraft, the question arises when is the most advantageous point to transition from one braking mode to the other. The relative magnitude of the forces involved are shown in figures 9.5 and 9.6.

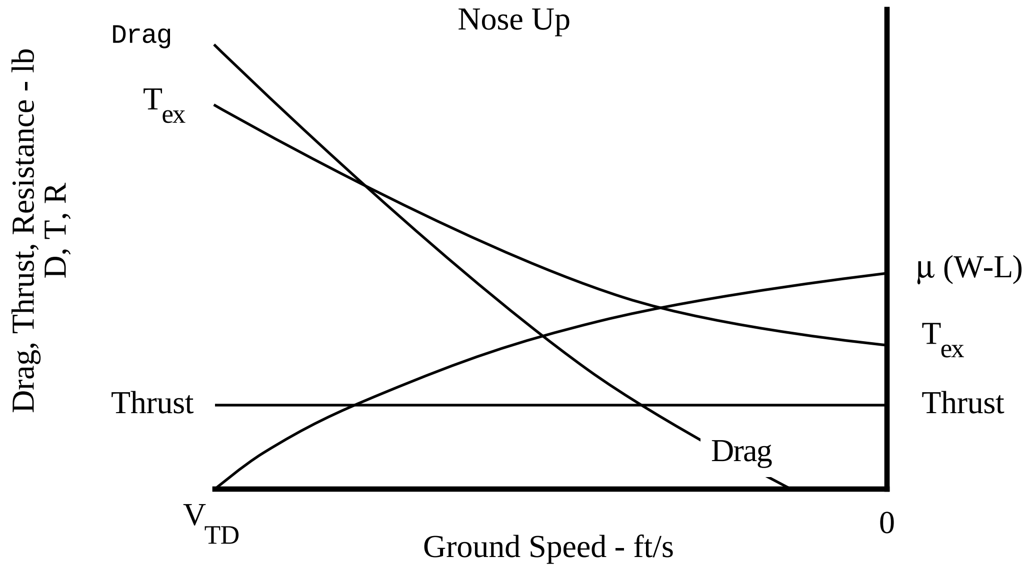


Figure 9.5

AERODYNAMIC BRAKING FORCES

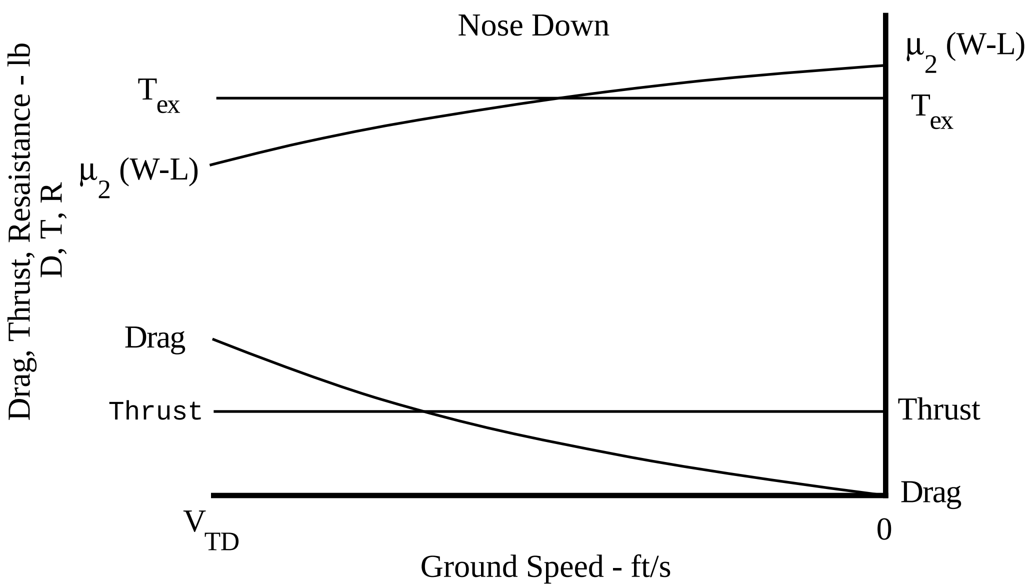
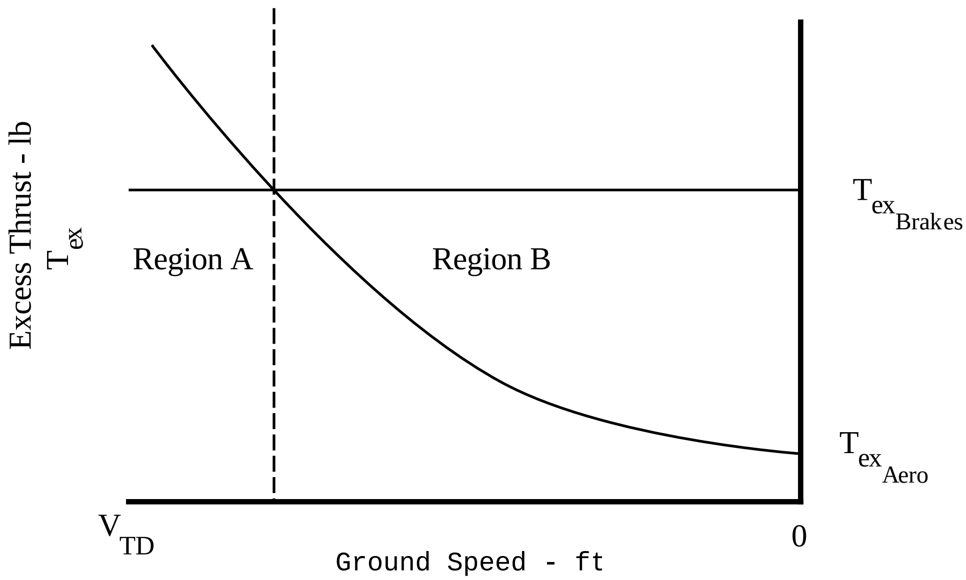


Figure 9.6

MAXIMUM WHEEL BRAKING FORCES

Notice that μ_2 (W-L) (where μ_2 is the coefficient of friction, brakes applied) is much greater than μ (W-L) which is the same as takeoff resistance. As shown in figure 9.7, the minimum stopping distance is achieved when aerodynamic braking is employed only as long as it provides a greater decelerating force than maximum wheel braking. An equation could be developed for the appropriate speed at which to make the transition using Eq 9.14 evaluated for both the aerodynamic braking and wheel braking condition. However, the resulting expression does not permit generalization of results.



Region A: Aerodynamic Braking Better
Region B: Wheel Braking Better

Figure 9.7
AERODYNAMIC AND WHEEL BRAKING FORCES

9.3.2.4 LANDING CORRECTIONS

The corrections to standard day conditions for landing data are similar to the methods used in the takeoff. The wind correction equation and the runway slope correction equation are identical to those applied to the takeoff performance. The equation for thrust, weight, and density is the same if reverse thrust is used, but may be simplified if idle thrust is used by setting the test thrust equal to the standard thrust. The relationships are:

Air phase:

$$S_{3_{Std}} = S_{3_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^{\left(2 + \frac{E_h}{E_h + 50}\right)} \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)^{\left(\frac{E_h}{E_h + 50}\right)} \quad \text{(Eq 9.35)}$$

In Eq 9.35, E_h is the energy height representing the kinetic energy change during the air phase, expressed as follows:

$$E_h = \frac{V_{50}^2 - V_{TD}^2}{2g} \quad \text{(Eq 9.36)}$$

Ground phase:

$$S_{4_{Std}} = S_{4_{Test}} \left(\frac{W_{Std}}{W_{Test}}\right)^2 \left(\frac{\sigma_{Test}}{\sigma_{Std}}\right) \quad \text{(Eq 9.37)}$$

Where:

E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
$S_{3_{Std}}$	Standard landing distance, 50 ft to touchdown	ft
$S_{3_{Test}}$	Test landing distance, 50 ft to touchdown	ft
$S_{4_{Std}}$	Standard landing distance, touchdown to stop	ft
$S_{4_{Test}}$	Test landing distance, touchdown to stop	ft
σ_{Std}	Standard density ratio	
σ_{Test}	Test density ratio	
V_{50}	Ground speed at 50 ft reference point	ft/s
V_{TD}	Touchdown ground speed	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

Past data has shown the weight correction to be valid for weights close to standard weight. In order to obtain data over a wide range of gross weights, a large number of tests must be conducted at carefully controlled weights at, or near, preselected standard values.

9.3.2.5 PILOT LANDING TECHNIQUE

Pilot technique is more important in the analysis of landing data than in takeoff data. Some of the factors which significantly affect landing performance include:

1. Power management during approach, flare, and touchdown.
2. Altitude of flare initiation.
3. Rate of rotation in the flare.
4. Length of hold-off time.
5. Touchdown speed.
6. Speed of braking initiation (aerodynamic and/or wheel) and brake pedal pressure.
7. Use of drag chute, spoilers, reverse thrust, or anti-skid.

9.4 TEST METHODS AND TECHNIQUES

9.4.1 TAKEOFF

9.4.1.1 TEST TECHNIQUE

To obtain repeatable takeoff performance data defining (and using) a repeatable takeoff technique is necessary.

1. Line up abeam a measured distance marker (runway remaining, Fresnel lens, etc.).
2. Ensure the nose wheel is straight.
3. Set takeoff power with engine stabilized (if possible), or establish throttle setting at/immediately after brake release.
4. Simultaneously release brakes and start clock or start clock and release brakes at a specified time.
5. Use rudder/nose wheel steering for alignment (no brakes).
6. Rotate at a prescribed airspeed to a specific attitude or angle of attack.

7. Once airborne, change configuration at specific altitude and airspeed.

While hand held data (stopwatch) can provide usable results, automatic recording devices are desired due to the dynamic nature of the tests.

9.4.1.2 DATA REQUIRED

1. Takeoff airspeed, V_{TO}.
2. Distance to lift off obtained by:
    a. Theodolite
    b. Runway camera.
    c. Paint gun.
    d. Observers.
    e. Laser.
    f. “Eyes right” - check runway marker.
3. Pitch attitude on rotation/initial climb.
4. Distance to 50 ft / distance to climb airspeed.
5. Time to lift off / 50 ft / climb airspeed.
6. Angle of attack at rotation/climb out.
7. Fuel used, brake release to climb airspeed.
8. Runway wind conditions.
9. Runway temperature.
10. Runway composition / runway condition reading (RCR).
11. Field elevation.
12. Altimeter.
13. Runway gradient.
14. Aircraft gross weight / center of gravity.
15. Power parameters: RPM, EGT.
16. Elevator/horizontal tail position.

9.4.1.3 TEST CRITERIA

1. Establish takeoff trim setting.
2. No brakes (limit use of rudder / nose wheel steering.).
3. Operate engine bleed air system OFF or in normal mode.

9.4.1.4 DATA REQUIREMENTS

1. Engine stabilized (if feasible).
2. Wind < 10 kn.
3. $W_{Test} \approx W_{Std}$.
4. Rate of climb from lift off to intercepting climb schedule < 500 ft/min.

9.4.1.5 SAFETY CONSIDERATIONS

1. Build up to define minimum lift off speeds / critical center of gravity etc.
2. Fly the aircraft first. Many parameters to observe/record. Set priorities.
3. Follow the course rules.
4. Don’t exceed the gear and flap speed limits.

9.4.2 LANDING

9.4.2.1 TEST TECHNIQUE

Use a repeatable defined technique for:

1. Approach (50 ft above the ground and flare point).
2. Flare (if required).
3. Touch down at a specific point (abeam Fresnel lens, etc.).
4. Aerodynamic braking (if appropriate) - specific attitude.
5. Braking (What speed? What pressure applied?).
6. Use of spoilers, thrust reverser(s), drag chute, anti-skid, etc.

While the landing distance can be measured by direct observation, automatic recording devices are desired (theodolite, runway camera etc.,) because of the dynamic nature of the tests.

9.4.2.2 DATA REQUIRED

1. Distance from 50 ft above the ground through the flare to touch down.
2. Glideslope angle.
3. Distance from touch down to full stop.
4. Runway wind conditions.
5. Runway temperature.
6. Runway composition / runway condition reading.
7. Field elevation.
8. Altimeter.
9. Runway gradient.
10. Aircraft gross weight / center of gravity.
11. Configuration.
12. Power parameters during landing and roll out: RPM, EGT.

9.4.2.3 TEST CRITERIA

Operate engine bleed air system OFF or in normal mode.

9.4.2.4 DATA REQUIREMENTS

1. Wind steady and < 10 kn.
2. $W_{Test} \approx W_{Std}$.

9.4.2.5 SAFETY CONSIDERATIONS

1. Build up to maximum effort stop landing (normally only demonstrated by the contractor).
2. Establish precautions against and procedures for:
    a. Hot brakes.
    b. Brake fire.
    c. Blown tire.
    d. Brake Failure.
3. Consider the geometry limit of aircraft for aerodynamic braking and sink rate at touch down.

9.5 DATA REDUCTION

The data for takeoff and landing performance is generally presented using test day weight and atmospheric conditions. The data reduction described below provides corrections to test day takeoff and landing distance to account for:

1. Wind.
2. Runway
3. Thrust, weight, and density.

9.5.1 TAKEOFF

From the pilot's data card and/or automatic recording device record:

1. Ground roll distance (brake release to lift off) (ft).
2. Wind velocity and direction relative to the runway (ft/s / degrees).
3. Lift off airspeed V_o (correct for position and instrument error (ft/s).
4. Temperature T_a (˚K).
5. Weight W (lb).
6. Pressure altitude H_P (ft).
7. Runway slope θ (deg).

The following equations are used in the data reduction:

$$V_w = \text{Wind Velocity } \cos(\text{Wind Direction Relative To Runway}) \qquad \text{(Eq 9.38)}$$

$$V_{TO_w} = V_{TO} - V_w \qquad \text{(Eq 9.19)}$$

$$S_{1_{Std}} = S_{1_w}\left(1 + \frac{V_w}{V_{TO_w}}\right)^{1.85} \qquad \text{(Eq 9.23)}$$

$$S_{1_{Std}} = \frac{S_{1_{SL}}}{\left(1 - \frac{2g\, S_{1_{SL}}}{V_{TO}^2}\sin\theta\right)} \qquad \text{(Eq 9.27)}$$

$$\sigma = 9.625\ \frac{P_a}{T_a} \qquad \text{(Eq 9.39)}$$

$$S_{1_{Std}} = S_{1_{Test}}\left(\frac{W_{Std}}{W_{Test}}\right)^{2.3}\left(\frac{\sigma_{Test}}{\sigma_{Std}}\right)\left(\frac{T_{N_{Test}}}{T_{N_{Std}}}\right)^{1.3} \qquad \text{(Eq 9.28)}$$

Where:

g	Gravitational acceleration	ft/s^2
P_a	Ambient pressure	psf
θ	Runway slope angle	deg
$S_{1_{SL}}$	Takeoff distance, brake release to lift off, sloping runway	ft
$S_{1_{Std}}$	Standard takeoff distance, brake release to lift off	ft
$S_{1_{Test}}$	Test takeoff distance, brake release to lift off	ft
S_{1_w}	Takeoff distance, brake release to lift off, with respect to wind	ft
σ_{Std}	Standard density ratio	
σ_{Test}	Test density ratio	
T_a	Ambient temperature	˚K
$T_{N_{Std}}$	Standard net thrust	lb
$T_{N_{Test}}$	Test net thrust	lb
V_{TO}	Takeoff ground speed	ft/s
V_{TO_w}	Takeoff ground speed with respect to wind	ft/s
V_w	Wind velocity	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

From the observed data compute as follows:

Step	Parameter	Notation	Formula	Units	Remarks
1	Wind component	V_w	Eq 9.38	ft/s	
2	Takeoff ground speed	V_{TO_w}	Eq 9.19	ft/s	
3	Ground roll	$S_{1_{Std}}$	Eq 9.23	ft	Wind corrected
4	Ground roll	$S_{1_{Std}}$	Eq 9.27	ft	Slope corrected
5	Density ratio	σ	Eq 9.39		
6	Ground roll	$S_{1_{Std}}$	Eq 9.28	ft	Thrust, weight, density corrected; T_N from thrust stand data

9.5.2 LANDING

From the pilot's data card and/or automatic recording device record:

1. Ground roll distance (touchdown to full stop) (ft).
2. Wind velocity and direction relative to the runway (ft/s / degrees).
3. Touchdown airspeed V_{TD} (correct for position and instrument error) (ft/s).
4. Temperature T_a (˚K).
5. Aircraft weight W (lb).
6. Pressure altitude H_P (ft).
7. Runway slope θ (deg).

The following equations are used in the data reduction.

$$V_w = \text{Wind Velocity } \cos(\text{Wind Direction Relative To Runway}) \qquad \text{(Eq 9.38)}$$

$$V_{TD_w} = V_{TD} - V_w \qquad \text{(Eq 9.40)}$$

$$S_{4_{Std}} = S_{4_w}\left(1 + \frac{V_w}{V_{TD}}\right)^{1.85} \qquad \text{(Eq 9.41)}$$

$$S_{4_{Std}} = \frac{S_{4_{SL}}}{\left(1 - \frac{2\,g\,S_{4_{SL}}}{V_{TD}^2}\sin\theta\right)} \tag{Eq 9.42}$$

$$\sigma = 9.625\,\frac{P_a}{T_a} \tag{Eq 9.39}$$

$$S_{4_{Std}} = S_{4_{Test}}\left(\frac{W_{Std}}{W_{Test}}\right)^2\left(\frac{\sigma_{Test}}{\sigma_{Std}}\right) \tag{Eq 9.37}$$

Where:

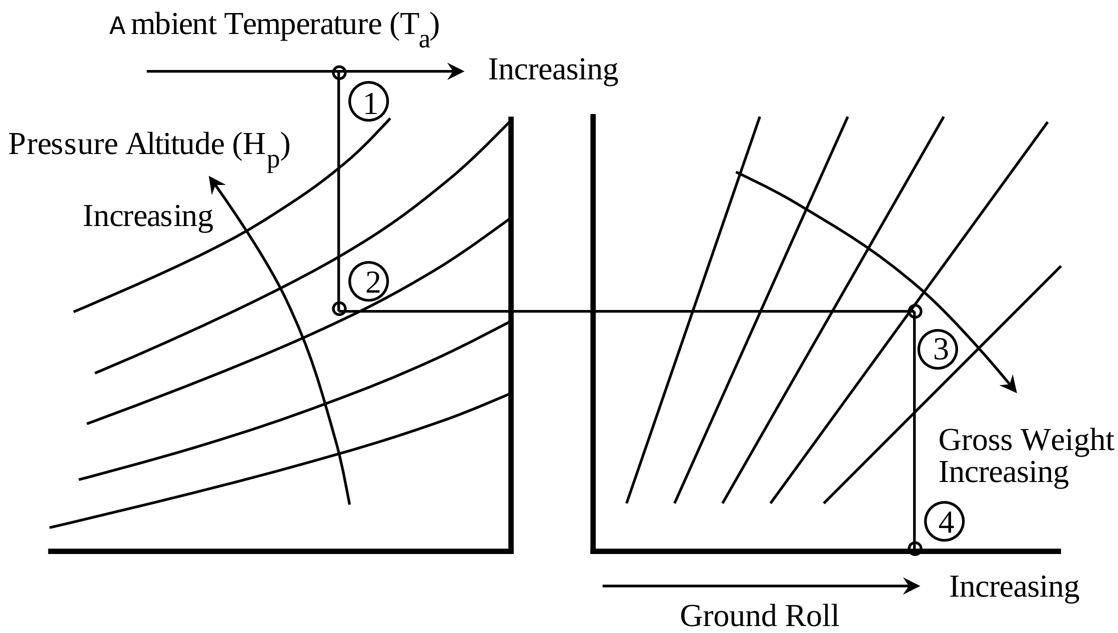
g	Gravitational acceleration	ft/s^2
P_a	Ambient pressure	psf
θ	Runway slope angle	deg
$S_{4_{SL}}$	Landing distance, touchdown to stop, sloping runway	ft
$S_{4_{Std}}$	Standard landing distance, touchdown to stop	ft
$S_{4_{Test}}$	Test landing distance, touchdown to stop	ft
S_{4_w}	Landing distance, touchdown to stop, with respect to wind	ft
σ_{Std}	Standard density ratio	
σ_{Test}	Test density ratio	
T_a	Ambient temperature	˚K
V_{TD}	Touchdown ground speed	ft/s
V_{TD_w}	Touchdown ground speed with respect to wind	ft/s
V_w	Wind velocity	ft/s
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb.

From the observed data compute as follows:

Step	Parameter	Notation	Formula	Units	Remarks
1	Wind component	V_w	Eq 9.38	ft/s	
2	Touchdown ground speed	V_{TD}	Eq 9.40	ft/s	
3	Ground roll	$S_{4_{Std}}$	Eq 9.41	ft	Wind corrected
4	Ground roll	$S_{4_{Std}}$	Eq 9.42	ft	Slope corrected
5	Density ratio	σ	Eq 9.39		
6	Ground roll	$S_{4_{Std}}$	Eq 9.37	ft	Weight, density corrected

9.6 DATA ANALYSIS

The analysis of takeoff and landing data is directed toward determining the optimum technique(s) to maximize the capabilities of the test aircraft. Once the data has been incorporated into figures similar to figures 9.8 and 9.9, takeoff ground roll can be determined for a given aircraft weight, ambient temperature, pressure altitude, wind, and runway slope.



1 Ambient Temperature

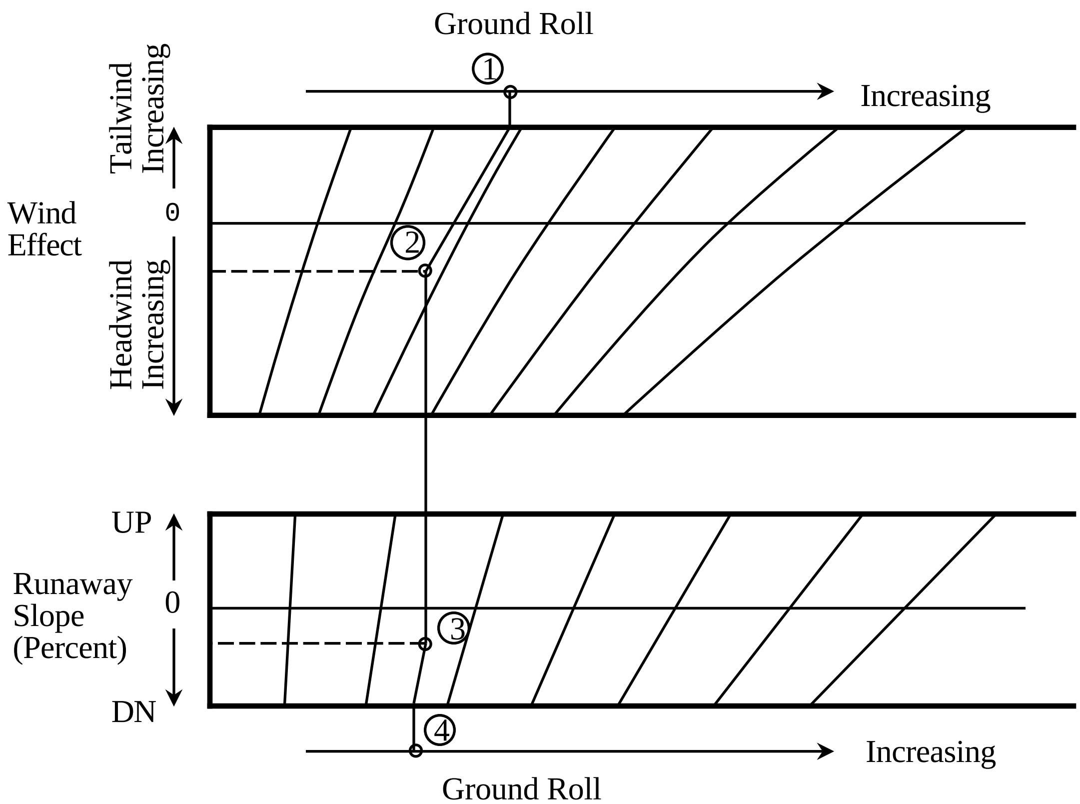
2 Pressure Altitude

3 Gross Weight

4 Ground Roll Distance

Figure 9.8

GROUND ROLL DISTANCE AS A FUNCTION OF TEMPERATURE, PRESSURE ALTITUDE, AND WEIGHT



1 Ground Roll From Figure 9.8

1 To 2 Wind Correction

2 To 3 Slope Correction

4 Adjusted Ground Roll

Figure 9.9
GROUND ROLL CORRECTIONS FOR WIND AND RUNWAY SLOPE

9.7 MISSION SUITABILITY

The requirements for takeoff and landing performance are specified in the detail specification for the aircraft. The determination of mission suitability depends largely on whether the aircraft meets those requirements.

9.8 SPECIFICATION COMPLIANCE

The takeoff and landing performance is normally covered as a contract guarantee in the detail specification requirements of each aircraft. For example, based on a standard day, takeoff configuration, and a specific drag index, the takeoff distance is specified to be not greater than a certain number of feet. Similarly, for the guaranteed landing performance at a specified gross weight, configuration, and braking condition, a distance not greater than a certain number of feet is specified.

9.9 GLOSSARY

9.9.1 NOTATIONS

AR	Aspect ratio	
C_D	Drag coefficient	
C_{D_i}	Induced drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
$C_{L_{max}}$	Maximum lift coefficient	
$CL_{max_{TO}}$	Maximum lift coefficient, takeoff configuration	
$C_{L_{Opt}}$	Optimum lift coefficient	
D	Drag	lb
ΔS_2	Change in S_2, equal to t V_w	ft
e	Oswald's efficiency factor	
E_h	Energy height	ft
g	Gravitational acceleration	ft/s^2
L	Lift	lb
N_{Std}	Standard propeller speed	rpm
N_{Test}	Test propeller speed	rpm
P_a	Ambient pressure	psf
$P_{a_{Std}}$	Standard ambient pressure	psf
$P_{a_{Test}}$	Test ambient pressure	psf
q	Dynamic pressure	psf
R	Resistance force	lb
S	Distance	ft
S	Wing area	ft^2

S_1	Takeoff distance, brake release to lift off	ft
$S_{1_{SL}}$	Takeoff distance, brake release to lift off, sloping runway	ft
$S_{1_{Std}}$	Standard takeoff distance, brake release to lift off	ft
$S_{1_{Test}}$	Test takeoff distance, brake release to lift off	ft
S_{1_w}	Takeoff distance, brake release to lift off, with respect to wind	ft
S_2	Takeoff distance, lift off to 50 ft	ft
$S_{2_{Std}}$	Standard takeoff distance, lift off to 50 ft	ft
$S_{2_{Test}}$	Test takeoff distance, lift off to 50 ft	ft
S_{2_w}	Takeoff distance, lift off to 50 ft, with respect to wind	ft
S_3	Landing distance, 50 ft to touchdown	ft
$S_{3_{Std}}$	Standard landing distance, 50 ft to touchdown	ft
$S_{3_{Test}}$	Test landing distance, 50 ft to touchdown	ft
S_4	Landing distance, touchdown to stop	ft
$S_{4_{SL}}$	Landing distance, touchdown to stop, sloping runway	ft
$S_{4_{Std}}$	Standard landing distance, touchdown to stop	ft
$S_{4_{Test}}$	Test landing distance, touchdown to stop	ft
S_{4_w}	Landing distance, touchdown to stop, with respect to wind	ft
T	Thrust	lb
t	Time	s
T_a	Ambient temperature	˚K
T_{ex}	Excess thrust	lb
$T_{ex_{Avg}}$	Average excess thrust	lb
$T_{ex_{Avg\ w}}$	Average excess thrust, with respect to wind	lb
T_N	Net thrust	lb
$T_{N_{Std}}$	Standard net thrust	lb
$T_{N_{Test}}$	Test net thrust	lb
V	Velocity	ft/s
V_{50}	Ground speed at 50 ft reference point	ft/s
$V_{CL_{50}}$	Climb speed at 50 ft	kn
V_L	Landing airspeed	ft/s, kn
$V_{L_{50}}$	Landing speed at 50 ft	kn

V_R	Rotation airspeed	kn
V_{s_L}	Stall speed, landing configuration, power off	kn
V_{s_T}	Stall speed, transition configuration, power off, flaps down, gear up	kn
V_{TD}	Touchdown ground speed	ft/s
V_{TD_w}	Touchdown ground speed with respect to wind	ft/s
V_{TO}	Takeoff ground speed	ft/s
V_{TO_w}	Takeoff ground speed with respect to wind	ft/s
V_w	Wind velocity	ft/s
W	Weight	lb
W_{Std}	Standard weight	lb
W_{Test}	Test weight	lb

9.9.2 GREEK SYMBOLS

μ (mu)	Coefficient of friction	
μ_2	Coefficient of friction, brakes applied	
π (pi)	Constant	
θ (theta)	Runway slope angle	deg
ρ (rho)	Air density	slugs/ft^3
σ_{Std} (sigma)	Standard density ratio	
σ_{Test}	Test density ratio	

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